Linear Trapezoidal Sallen Key Filter (SKF) in state increment form: state += val

created: 18th Dec 2014
last updated: 1st Feb 2016 (added explicit notch and peak responses to psuedo code)

Putting it in the form of a state increment keeps the all coefficients near zero at low frequencies, which is feels intuitively right to me since you just increment a bit of signal onto your existing state to do your integration.

I haven't had time to add too much detail here, but please refer to the other papers located at www.cytomic.com/technical-papers for derivations of trapezoidal integration and the maths of circuit solving.

Thanks to Teemu Voipio (mystran) for pointing out the numerical benefits of using sin to calcualte the coeffecients.

Low pass SKF

Solve the nodal circuit equations for an implicit linear SKF, low only

```
Remove["Global`*"]
sln = Solve[{v1 == (k) v2 + vc1, va1 == (v0 - v1), v2 == 0 + vc2, va2 == (v1 - v2),
     vc1 == ic1eq + g va1, vc2 == ic2eq + g va2}, {vc1, vc2}, {va1, va2, v1, v2}][[1]]
sln1 = a /. Solve[(vc1 /. sln) == ic1eq + a, a][[1]];
sln2 = a /. Solve[(vc2 /. sln) == ic2eq + a, a][[1]];
{vc1 → ic1eq + sln1}
{vc2 → ic2eq + sln2}
coef1 = Coefficient[sln1, {v0, ic1eq, ic2eq}] // Simplify
coef2 = Coefficient[sln2, {v0, ic1eq, ic2eq}] // Simplify
```

$$\left\{vc1 \to -\left(\left(-ic1eq - g\, ic1eq + g\, ic1eq\, k + g\, ic2eq\, k - g\, v0 - g^2\, v0 + g^2\, k\, v0\right) \Big/ \left(1 + 2\, g + g^2 - g\, k\right)\right),\ vc2 \to -\frac{-g\, ic1eq - ic2eq - g\, ic2eq - g^2\, v0}{1 + 2\, g + g^2 - g\, k}\right\}$$

$$\left\{vc1 \to ic1eq + \frac{-g\, ic1eq - g^2\, ic1eq - g\, ic2eq\, k + g\, v0 + g^2\, v0 - g^2\, k\, v0}{1 + 2\, g + g^2 - g\, k}\right\}$$

$$\left\{vc2 \to ic2eq + \frac{g\, ic1eq - g\, ic2eq - g^2\, ic2eq + g\, ic2eq\, k + g^2\, v0}{1 + 2\, g + g^2 - g\, k}\right\}$$

$$\left\{\frac{g\,(1 + g - g\, k)}{1 + g^2 - g\,(-2 + k)},\ -\frac{g\,(1 + g)}{1 + g^2 - g\,(-2 + k)},\ -\frac{g\, k}{1 + 2\, g + g^2 - g\, k}\right\}$$

$$\left\{\frac{g^2}{1 + 2\, g + g^2 - g\, k},\ \frac{g}{1 + 2\, g + g^2 - g\, k},\ \frac{g\,(-1 - g + k)}{1 + g^2 - g\,(-2 + k)}\right\}$$

Coefficients using Tan, low only

```
gt = {g0 → coef1[[1]] /. {g → Tan[π w]}, g1 → coef1[[2]] /. {g → Tan[π w]},
   g2 → coef1[[3]] /. {g → Tan[π w]}, g3 → coef2[[1]] /. {g → Tan[π w]},
   g4 → coef2[[2]] /. {g → Tan[π w]}, g5 → coef2[[3]] /. {g → Tan[π w]}}
```

$$\left\{g0 \to \frac{\mathrm{Tan}[\pi\, w]\,(1 + \mathrm{Tan}[\pi\, w] - k\, \mathrm{Tan}[\pi\, w])}{1 - (-2 + k)\, \mathrm{Tan}[\pi\, w] + \mathrm{Tan}[\pi\, w]^2},\ g1 \to -\frac{\mathrm{Tan}[\pi\, w]\,(1 + \mathrm{Tan}[\pi\, w])}{1 - (-2 + k)\, \mathrm{Tan}[\pi\, w] + \mathrm{Tan}[\pi\, w]^2},\right.$$
$$g2 \to -\frac{k\, \mathrm{Tan}[\pi\, w]}{1 + 2\, \mathrm{Tan}[\pi\, w] - k\, \mathrm{Tan}[\pi\, w] + \mathrm{Tan}[\pi\, w]^2},\ g3 \to \frac{\mathrm{Tan}[\pi\, w]^2}{1 + 2\, \mathrm{Tan}[\pi\, w] - k\, \mathrm{Tan}[\pi\, w] + \mathrm{Tan}[\pi\, w]^2},$$
$$\left.g4 \to \frac{\mathrm{Tan}[\pi\, w]}{1 + 2\, \mathrm{Tan}[\pi\, w] - k\, \mathrm{Tan}[\pi\, w] + \mathrm{Tan}[\pi\, w]^2},\ g5 \to \frac{(-1 + k - \mathrm{Tan}[\pi\, w])\, \mathrm{Tan}[\pi\, w]}{1 - (-2 + k)\, \mathrm{Tan}[\pi\, w] + \mathrm{Tan}[\pi\, w]^2}\right\}$$

Coefficients in terms of Sin instead of Tan, low only

```
ToSins1[x_] := Block[{ss},
   ss = ((ExpandNumerator[ExpandDenominator[FullSimplify[x /. {g → s / c}]]] //.
         {c s :→ 1 / 2 s2}) //. {c^2 :→ 1 - s^2}) /.
     {c → Cos[π w], s → Sin[π w], s2 → Sin[2 π w]};
   ss = Simplify[Numerator[ss], ExcludedForms → {Sin[_]}] /
     Simplify[Denominator[ss], ExcludedForms → {Sin[_]}];
   Return[ss];
  ];
gs = {g0 → ToSins1[coef1[[1]]], g1 → ToSins1[coef1[[2]]],
   g2 → ToSins1[coef1[[3]]], g3 → ToSins1[coef2[[1]]],
   g4 → ToSins1[coef2[[2]]], g5 → ToSins1[coef2[[3]]]}
```

$$\left\{g0 \to \frac{-(-1+k)\,\mathrm{Sin}[\pi w]^2 + \frac{1}{2}\mathrm{Sin}[2\pi w]}{1+\mathrm{Sin}[2\pi w] - \frac{1}{2} k\,\mathrm{Sin}[2\pi w]},\ g1 \to \frac{-\mathrm{Sin}[\pi w]^2 - \frac{1}{2}\mathrm{Sin}[2\pi w]}{1+\mathrm{Sin}[2\pi w] - \frac{1}{2} k\,\mathrm{Sin}[2\pi w]},\right.$$
$$g2 \to -\frac{k\,\mathrm{Sin}[2\pi w]}{2-(-2+k)\,\mathrm{Sin}[2\pi w]},\ g3 \to \frac{\mathrm{Sin}[\pi w]^2}{1+\mathrm{Sin}[2\pi w] - \frac{1}{2} k\,\mathrm{Sin}[2\pi w]},$$
$$\left. g4 \to \frac{\mathrm{Sin}[2\pi w]}{2-(-2+k)\,\mathrm{Sin}[2\pi w]},\ g5 \to \frac{2\,\mathrm{Sin}[\pi w]^2 + \mathrm{Sin}[2\pi w] - k\,\mathrm{Sin}[2\pi w]}{-2+(-2+k)\,\mathrm{Sin}[2\pi w]}\right\}$$

Manual simplification of the Sin only terms, low only

$$\mathbf{gsm} = \left\{g0 \to \frac{2\,(1-k)\,\mathrm{Sin}[\pi w]^2 + \mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\ g1 \to \frac{-2\,\mathrm{Sin}[\pi w]^2 - \mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\right.$$
$$g2 \to \frac{-k\,\mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\ g3 \to \frac{2\,\mathrm{Sin}[\pi w]^2}{2+(2-k)\,\mathrm{Sin}[2\pi w]},$$
$$\left. g4 \to \frac{\mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\ g5 \to \frac{-2\,\mathrm{Sin}[\pi w]^2 - (1-k)\,\mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]}\right\}$$

```
FullSimplify[gs - gsm]
```

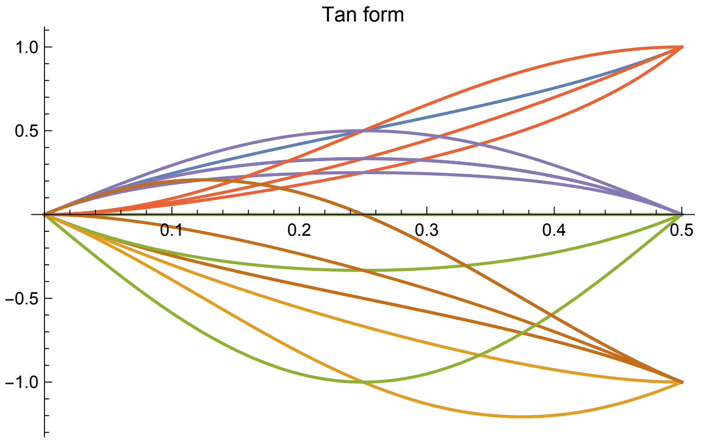
$$\left\{g0 \to \frac{2\,(1-k)\,\mathrm{Sin}[\pi w]^2 + \mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\ g1 \to \frac{-2\,\mathrm{Sin}[\pi w]^2 - \mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\right.$$
$$g2 \to -\frac{k\,\mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\ g3 \to \frac{2\,\mathrm{Sin}[\pi w]^2}{2+(2-k)\,\mathrm{Sin}[2\pi w]},$$
$$\left. g4 \to \frac{\mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]},\ g5 \to \frac{-2\,\mathrm{Sin}[\pi w]^2 - (1-k)\,\mathrm{Sin}[2\pi w]}{2+(2-k)\,\mathrm{Sin}[2\pi w]}\right\}$$

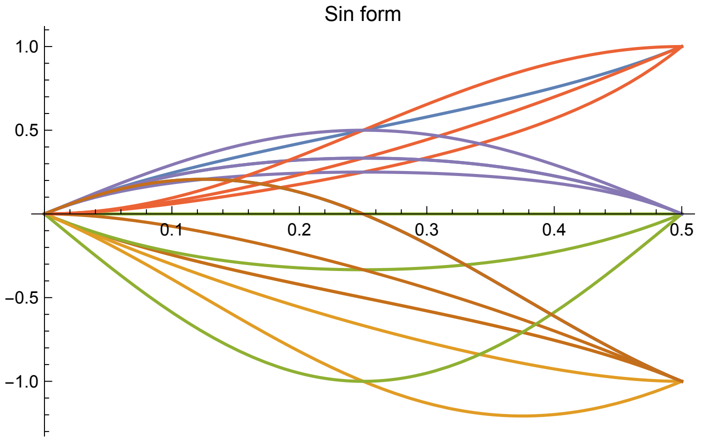
{0, 0, 0, 0, 0, 0}

Plot the coefficients to check the sin version matches the tan one

Note that the coefficients are near zero when the cutoff is near zero, and they remain bounded

```
Show[Table[Plot[{g0 /. gt, g1 /. gt, g2 /. gt, g3 /. gt, g4 /. gt, g5 /. gt},
    {w, 0, 1 / 2}], {k, 0, 2}], PlotLabel → "Tan form", PlotRange → All]

Show[Table[Plot[{g0 /. gsm, g1 /. gsm, g2 /. gsm, g3 /. gsm, g4 /. gsm, g5 /. gsm},
    {w, 0, 1 / 2}], {k, 0, 2}], PlotLabel → "Sin form", PlotRange → All]
```





Psuedo code, low only

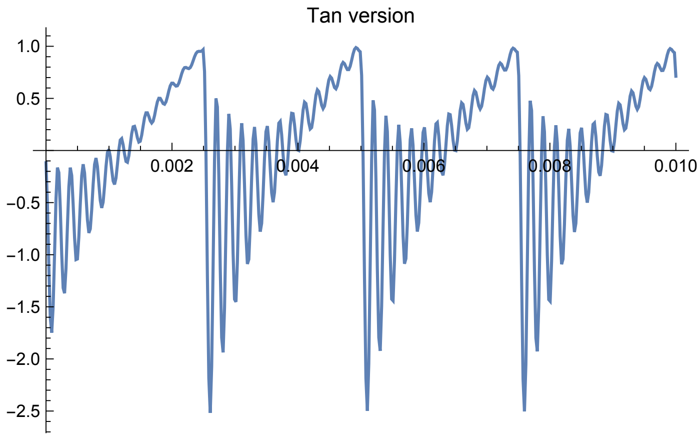
```
init :
k = 2*res
w = pi*cutoff/samplerate
s1 = sin (w)
s2 = sin (2*w)
nrm = 1/(2 + (2 - k)*s2)
s1n = 2*s1*s1*nrm
s2n = s2*nrm
g0 = (1 - k)*s1n + s2n
g1 = -s1n - s2n
g2 = -k*s2n
g3 = s1n
g4 = s2n
g5 = -s1n - (1 - k)*s2n

clear :
ic1eq = 0
ic2eq = 0

tick :
v0 = input
t1 = g0*v0 + g1*ic1eq + g2*ic2eq
t2 = g3*v0 + g4*ic1eq + g5*ic2eq
vc1 = t1 + ic1eq (not needed)
vc2 = t2 + ic2eq
ic1eq = ic1eq + 2.0*t1
```

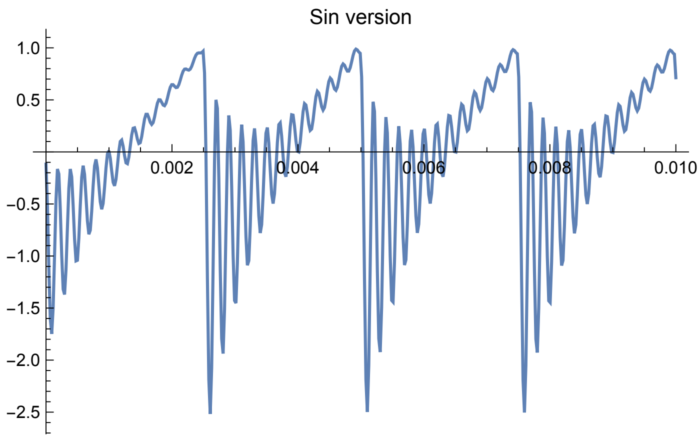
```
ic2eq = ic2eq + 2.0*t2
```

Implementation check against tan version, low only

```
CalcCoeff1[cutoff_, res_, sr_] := Block[{w, nrm},
    w = cutoff / sr;
    k = res;
    g = Tan[π w];
  ];
CalcCoeff2[cutoff_, res_, sr_] := Block[{w, nrm, s1, s2, s2n, ssn},
    w = cutoff / sr;
    k = res;
    s1 = Sin[π w];
    s2 = Sin[2 π w];
    w = π * cutoff / sr;
    s1 = Sin[w];
    s2 = Sin[2 * w];
    nrm = 1 / (2 + (2 - k) * s2);
    s1n = 2 * s1 * s1 * nrm;
    s2n = s2 * nrm;
    g0 = (1 - k) * s1n + s2n;
    g1 = -s1n - s2n;
    g2 = -k * s2n;
    g3 = s1n;
    g4 = s2n;
    g5 = -s1n - (1 - k) * s2n;
  ];
Tick1[t_, v0_] :=
  Block[{v1, v2, t1, t2, high, band, low, peak, notch, vc1, vc2},
    vc1 = -(-g ic2eq k - (-1 - g + g k) (ic1eq + g v0))/(-1 - 2 g - g^2 + g k);
    vc2 = -(-g ic1eq - ic2eq - g ic2eq - g^2 v0)/(1 + 2 g + g^2 - g k);
    ic1eq = 2 vc1 - ic1eq;
    ic2eq = 2 vc2 - ic2eq;
    low = vc2;
    Return[{t, v0, low}]
  ];
Tick2[t_, v0_] :=
  Block[{v1, v2, t0, tc1, t1, t2, high, band, low, peak, notch, vc1, vc2},
    t1 = g0 * v0 + g1 * ic1eq + g2 * ic2eq;
    t2 = g3 * v0 + g4 * ic1eq + g5 * ic2eq;
    vc1 = ic1eq + t1;
    vc2 = ic2eq + t2;
    ic1eq = ic1eq + 2.0 * t1;
    ic2eq = ic2eq + 2.0 * t2;
    low = vc2;
    Return[{t, v0, low}]
  ];

sr = 44 100.0;
h = 1.0 / sr;
cutoff = 5000.0;
res = 1.9;

MySaw[x_] := 2 (x - Floor[x] - 0.5);
```

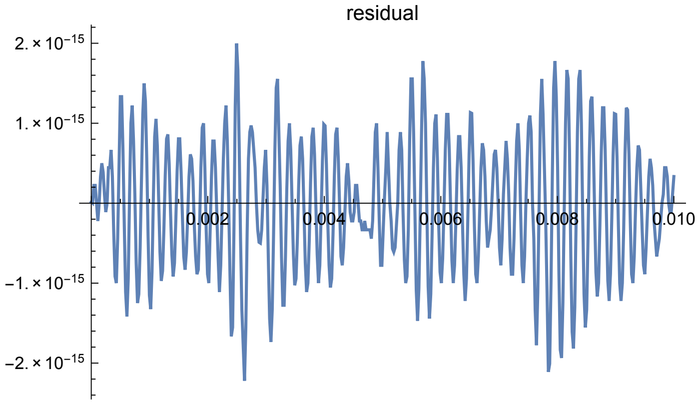
```
MyOsc[x_] := 1 MySaw[400 x];

CalcCoeff1[cutoff, res, sr];
CalcCoeff2[cutoff, res, sr];
ic1eq = ic2eq = 0;
tp1 = Table[Tick1[t, MyOsc[t]] , {t, 0, 0.01, h}];
ic1eq = ic2eq = 0;
tp2 = Table[Tick2[t, MyOsc[t]] , {t, 0, 0.01, h}];
ListPlot[Table[tp1[[All, {1, i}]], {i, 3, 3}],
 Joined → True, PlotRange → All, PlotLabel → "Tan version"]
ListPlot[Table[tp2[[All, {1, i}]], {i, 3, 3}],
 Joined → True, PlotRange → All, PlotLabel → "Sin version"]
```





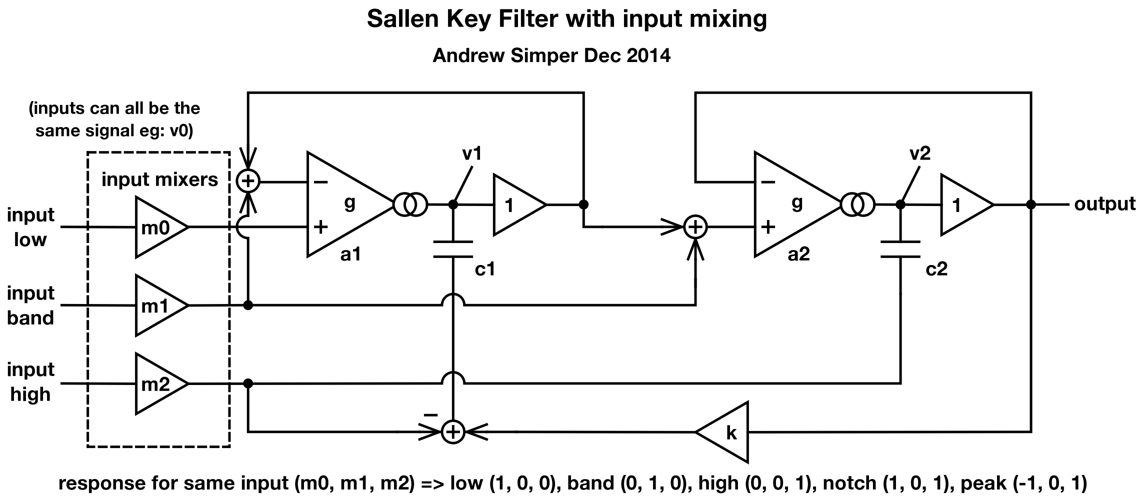
This residual shows that the Sin version is slightly more accurate than the Tan version, which will be more important for limited precision implementations

```
tp3 = Table[Flatten[{tp1[[j, 1]], tp1[[j, 2]],
      Table[tp1[[j, i]] - tp2[[j, i]], {i, 3, 3}]}], {j, 1, Length[tp1]}];
ListPlot[Table[tp3[[All, {1, i}]], {i, 3, 3}], Joined → True,
 PlotRange → All, PlotLabel → "residual"]
```



Mixing inputs to make low, band, high SKF

Note : you can make notch and peak by summing or differencing low and high like you do with an SVF



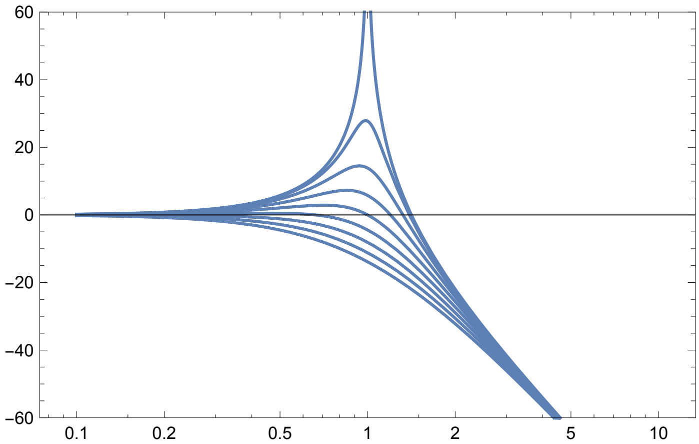
Solve the laplace equations

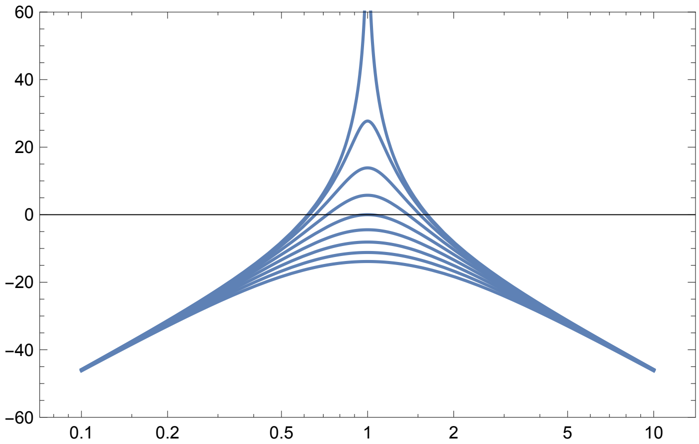
```
Remove["Global`*"]
eqn = {va1 == (m0 v0 - m1 v0 - v1), va2 == (m1 v0 + v1 - v2), v1 == (k) v2 - m2 v0 + vc1,
   v2 == m2 v0 + vc2, 0 == g va1 - s vc1, 0 == g va2 - s vc2, hs == v2 / v0};
hs = hs /. Solve[eqn, {hs}, {v1, v2, va1, va2, vc1, vc2}][[1]] // FullSimplify
```

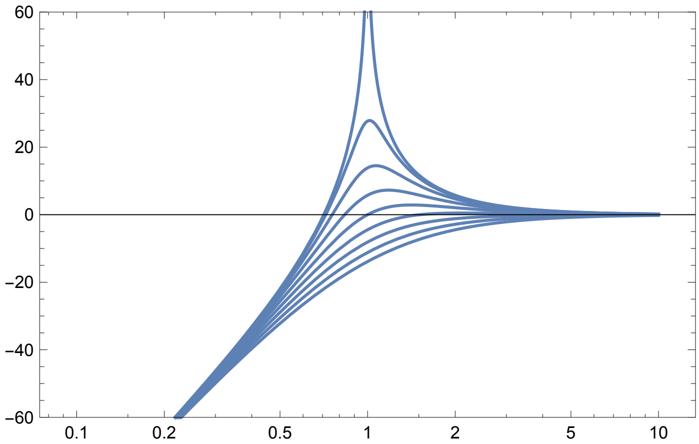
$$\frac{g^2\,m0 + g\,m1\,s + m2\,s^2}{g^2 - g\,(-2 + k)\,s + s^2}$$

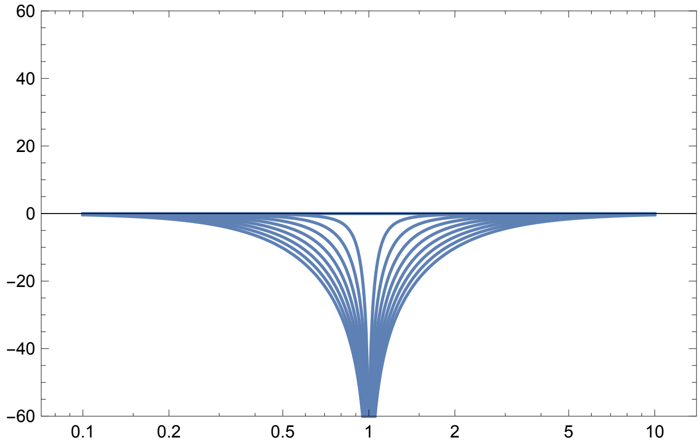
Boring plots of amplitude response, yes it is a low, band, high, notch, and peaking filter!

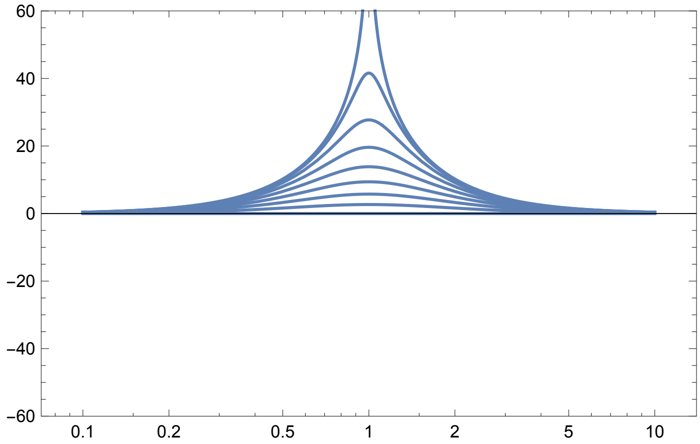
```
dB[x_] := 20 Log[Abs[x]]
AmplitudePlot[hs_, m_] := Show[Table[LogLinearPlot[
     dB[hs /. {m0 → m[[1]], m1 → m[[2]], m2 → m[[3]], s → ⅈ w, g → 1}],
     {w, 0.1, 10}, PlotRange → {60, -60}, Frame → True], {k, 0, 2, 0.25}]];
AmplitudePlot[hs, {1, 0, 0}]
AmplitudePlot[hs, {0, 1, 0}]
AmplitudePlot[hs, {0, 0, 1}]
AmplitudePlot[hs, {1, 0, 1}]
AmplitudePlot[hs, {-1, 0, 1}]
```








60
40
20
0
−20
−40
−60
0.1
0.2
0.5
1
2
5
10


60
40
20
0
−20
−40
−60
0.1
0.2
0.5
1
2
5
10

Solve the nodal circuit equations for an implicit linear SKF, mixing for low, band, high

```
Remove["Global`*"]
sln =
 Solve[{va1 == (m0 v0 - m1 v0 - v1), va2 == (m1 v0 + v1 - v2), v1 == (k) v2 - m2 v0 + vc1,
     v2 == m2 v0 + vc2, 0 == g va1 - (vc1 - ic1eq), 0 == g va2 - (vc2 - ic2eq)},
    {vc1, vc2}, {v1, v2, va1, va2}][[1]] // FullSimplify
sln1 = a /. Solve[(vc1 /. sln) == ic1eq + a, a][[1]];
sln2 = a /. Solve[(vc2 /. sln) == ic2eq + a, a][[1]];
{vc1 → ic1eq + sln1}
{vc2 → ic2eq + sln2}
coef1 = Coefficient[sln1, {v0, ic1eq, ic2eq}] // Simplify
coef2 = Coefficient[sln2, {v0, ic1eq, ic2eq}] // Simplify
```

$$\left\{vc1\to\frac{1}{(1+g)^2-g\,k}\left(-g\,ic2eq\,k+ic1eq\,(1+g-g\,k)+g\,((1+g-g\,k)\,m0-(1+g)\,m1+(1+g-k)\,m2)\,v0\right),\ vc2\to\frac{1}{(1+g)^2-g\,k}\left(ic2eq+g\left(ic1eq+ic2eq+(g\,m0+m1+(-2-g+k)\,m2)\,v0\right)\right)\right\}$$

$$\{vc1\to ic1eq+\left(-g\,ic1eq-g^2\,ic1eq-g\,ic2eq\,k+g\,m0\,v0+g^2\,m0\,v0-g^2\,k\,m0\,v0-g\,m1\,v0-g^2\,m1\,v0+g\,m2\,v0+g^2\,m2\,v0-g\,k\,m2\,v0\right)/\left(1+2\,g+g^2-g\,k\right)\}$$

$$\{vc2\to ic2eq+\left(g\,ic1eq-g\,ic2eq-g^2\,ic2eq+g\,ic2eq\,k+g^2\,m0\,v0+g\,m1\,v0-2\,g\,m2\,v0-g^2\,m2\,v0+g\,k\,m2\,v0\right)/\left(1+2\,g+g^2-g\,k\right)\}$$

$$\left\{\frac{g\,((1+g-g\,k)\,m0-(1+g)\,m1+(1+g-k)\,m2)}{1+g^2-g\,(-2+k)},\ -\frac{g\,(1+g)}{1+g^2-g\,(-2+k)},\ -\frac{g\,k}{1+2\,g+g^2-g\,k}\right\}$$

$$\left\{\frac{g\,(m1+g\,(m0-m2)+(-2+k)\,m2)}{1+g^2-g\,(-2+k)},\ \frac{g}{1+2\,g+g^2-g\,k},\ \frac{g\,(-1-g+k)}{1+g^2-g\,(-2+k)}\right\}$$

Coefficients using Tan, mixing for low, band, high

```
gt = {g0 → coef1[[1]] /. {g → Tan[π w]}, g1 → coef1[[2]] /. {g → Tan[π w]},
   g2 → coef1[[3]] /. {g → Tan[π w]}, g3 → coef2[[1]] /. {g → Tan[π w]},
   g4 → coef2[[2]] /. {g → Tan[π w]}, g5 → coef2[[3]] /. {g → Tan[π w]}}
```

$$\{g0\to(\mathrm{Tan}[\pi\,w]\,(-m1\,(1+\mathrm{Tan}[\pi\,w])+m2\,(1-k+\mathrm{Tan}[\pi\,w])+m0\,(1+\mathrm{Tan}[\pi\,w]-k\,\mathrm{Tan}[\pi\,w])))/\left(1-(-2+k)\,\mathrm{Tan}[\pi\,w]+\mathrm{Tan}[\pi\,w]^2\right),$$

$$g1\to-\frac{\mathrm{Tan}[\pi\,w]\,(1+\mathrm{Tan}[\pi\,w])}{1-(-2+k)\,\mathrm{Tan}[\pi\,w]+\mathrm{Tan}[\pi\,w]^2},\ g2\to-\frac{k\,\mathrm{Tan}[\pi\,w]}{1+2\,\mathrm{Tan}[\pi\,w]-k\,\mathrm{Tan}[\pi\,w]+\mathrm{Tan}[\pi\,w]^2},$$

$$g3\to\frac{\mathrm{Tan}[\pi\,w]\,(m1+(-2+k)\,m2+(m0-m2)\,\mathrm{Tan}[\pi\,w])}{1-(-2+k)\,\mathrm{Tan}[\pi\,w]+\mathrm{Tan}[\pi\,w]^2},$$

$$g4\to\frac{\mathrm{Tan}[\pi\,w]}{1+2\,\mathrm{Tan}[\pi\,w]-k\,\mathrm{Tan}[\pi\,w]+\mathrm{Tan}[\pi\,w]^2},$$

$$g5\to\frac{(-1+k-\mathrm{Tan}[\pi\,w])\,\mathrm{Tan}[\pi\,w]}{1-(-2+k)\,\mathrm{Tan}[\pi\,w]+\mathrm{Tan}[\pi\,w]^2}\}$$

Coefficients in terms of Sin instead of Tan, mixing for low, band, high

```
ToSins1[x_] := Block[{ss},
   ss = ((ExpandNumerator[ExpandDenominator[FullSimplify[x /. {g → s / c}]]] //.
          {c s :→ 1 / 2 s2}) //. {c^2 :→ 1 - s^2}) /.
     {c → Cos[π w], s → Sin[π w], s2 → Sin[2 π w]};
   ss = Simplify[Numerator[ss], ExcludedForms → {Sin[_]}] /
     Simplify[Denominator[ss], ExcludedForms → {Sin[_]}];
   Return[ss];
  ];
gs = {g0 → ToSins1[coef1[[1]]], g1 → ToSins1[coef1[[2]]],
   g2 → ToSins1[coef1[[3]]], g3 → ToSins1[coef2[[1]]],
   g4 → ToSins1[coef2[[2]]], g5 → ToSins1[coef2[[3]]]}
```

$$\left\{g0 \to \left(-m1\left(2\,\text{Sin}[\pi w]^2+\text{Sin}[2\pi w]\right)+m0\left(-2\,(-1+k)\,\text{Sin}[\pi w]^2+\text{Sin}[2\pi w]\right)+m2\left(2\,\text{Sin}[\pi w]^2+\text{Sin}[2\pi w]-k\,\text{Sin}[2\pi w]\right)\right)\Big/\left(2\left(1+\text{Sin}[2\pi w]-\frac{1}{2}k\,\text{Sin}[2\pi w]\right)\right),\right.$$

$$g1 \to \frac{-\text{Sin}[\pi w]^2-\frac{1}{2}\text{Sin}[2\pi w]}{1+\text{Sin}[2\pi w]-\frac{1}{2}k\,\text{Sin}[2\pi w]},\; g2 \to -\frac{k\,\text{Sin}[2\pi w]}{2-(-2+k)\,\text{Sin}[2\pi w]},$$

$$g3 \to \left(2\,m0\,\text{Sin}[\pi w]^2+m1\,\text{Sin}[2\pi w]+m2\left(-2\,\text{Sin}[\pi w]^2-2\,\text{Sin}[2\pi w]+k\,\text{Sin}[2\pi w]\right)\right)\Big/\left(2\left(1+\text{Sin}[2\pi w]-\frac{1}{2}k\,\text{Sin}[2\pi w]\right)\right),$$

$$\left. g4 \to \frac{\text{Sin}[2\pi w]}{2-(-2+k)\,\text{Sin}[2\pi w]},\; g5 \to \frac{2\,\text{Sin}[\pi w]^2+\text{Sin}[2\pi w]-k\,\text{Sin}[2\pi w]}{-2+(-2+k)\,\text{Sin}[2\pi w]}\right\}$$

Manual simplification of the Sin only terms, mixing for low, band, high

```
gsm = {g0 → (m0 ((1 - k) 2 Sin[π w]^2 + Sin[2 π w]) + m1 (-2 Sin[π w]^2 - Sin[2 π w]) +
        m2 (2 Sin[π w]^2 + (1 - k) Sin[2 π w])) / (2 + (2 - k) Sin[2 π w]),
   g1 → (-2 Sin[π w]^2 - Sin[2 π w]) / (2 + (2 - k) Sin[2 π w]), g2 → (-k Sin[2 π w]) / (2 + (2 - k) Sin[2 π w]),
   g3 → ( m0 (2 Sin[π w]^2) + m1 (Sin[2 π w]) + m2 (-2 Sin[π w]^2 - (2 - k) Sin[2 π w])) /
     (2 + (2 - k) Sin[2 π w]),
   g4 → Sin[2 π w] / (2 + (2 - k) Sin[2 π w]), g5 → (-2 Sin[π w]^2 - (1 - k) Sin[2 π w]) / (2 + (2 - k) Sin[2 π w])}
FullSimplify[gs - gsm]
```

$$\left\{g0 \to \left(m1\left(-2\,\text{Sin}[\pi w]^2-\text{Sin}[2\pi w]\right)+m0\left(2\,(1-k)\,\text{Sin}[\pi w]^2+\text{Sin}[2\pi w]\right)+m2\left(2\,\text{Sin}[\pi w]^2+(1-k)\,\text{Sin}[2\pi w]\right)\right)\Big/\left(2+(2-k)\,\text{Sin}[2\pi w]\right),\right.$$

$$g1 \to \frac{-2\,\text{Sin}[\pi w]^2-\text{Sin}[2\pi w]}{2+(2-k)\,\text{Sin}[2\pi w]},\; g2 \to -\frac{k\,\text{Sin}[2\pi w]}{2+(2-k)\,\text{Sin}[2\pi w]},$$

$$g3 \to \left(2\,m0\,\text{Sin}[\pi w]^2+m1\,\text{Sin}[2\pi w]+m2\left(-2\,\text{Sin}[\pi w]^2-(2-k)\,\text{Sin}[2\pi w]\right)\right)\Big/\left(2+(2-k)\,\text{Sin}[2\pi w]\right),$$

$$\left. g4 \to \frac{\text{Sin}[2\pi w]}{2+(2-k)\,\text{Sin}[2\pi w]},\; g5 \to \frac{-2\,\text{Sin}[\pi w]^2-(1-k)\,\text{Sin}[2\pi w]}{2+(2-k)\,\text{Sin}[2\pi w]}\right\}$$

{0, 0, 0, 0, 0, 0}

Psuedo code, mixing for low, band, high, identical input

```
init :
m0 = low mix
```

```
m1 = band mix
m2 = high mix
for notch use m2 = m0
for peak use m2 = -m0
k = 2*res
w = pi*cutoff/samplerate
s1 = sin (w)
s2 = sin (2*w)
nrm = 1/(2 + (2 - k)*s2)
s1n = 2*s1*s1*nrm
s2n = s2*nrm
g0 = m0*((1 - k)*s1n + s2n) + m1*(-s1n - s2n) + m2*(s1n + (1 - k)*s2n)
g1 = -s1n - s2n
g2 = -k*s2n
g3 = m0*(s1n) + m1*(s2n) + m2*(-s1n - (2 - k)*s2n)
g4 = s2n
g5 = -s1n - (1 - k)*s2n

clear :
ic1eq = 0
ic2eq = 0

tick :
v0 = input
t1 = g0*v0 + g1*ic1eq + g2*ic2eq
t2 = g3*v0 + g4*ic1eq + g5*ic2eq
vc1 = t1 + ic1eq (not needed)
vc2 = t2 + ic2eq
ic1eq = ic1eq + 2.0*t1
ic2eq = ic2eq + 2.0*t2
```

Psuedo code, mixing for low, band, high, decoupling mixing from coeffs, possibly different inputs

```
init :
m0 = low mix
m1 = band mix
m2 = high mix
for notch use m2 = m0
for peak use m2 = -m0
vlow = low pass input*m0
vband = band pass input*m1
vhigh = high pass input*m2
for notch and peak use high pass input = low pass input
k = 2*res
w = pi*cutoff/samplerate
s1 = sin (w)
s2 = sin (2*w)
nrm = 1/(2 + (2 - k)*s2)
s1n = 2*s1*s1*nrm
s2n = s2*nrm
g0m0 = (1 - k)*s1n + s2n
g0m1 = -s1n - s2n
g0m2 = s1n + (1 - k)*s2n
g1 = -s1n - s2n
g2 = -k*s2n
g3m0 = s1n
g3m1 = s2n
g3m2 = -s1n - (2 - k)*s2n
g4 = s2n
g5 = -s1n - (1 - k)*s2n

clear :
ic1eq = 0
ic2eq = 0

tick :
v0 = input
t1 = vlow*g0m0 + vband*g0m1 + vhigh*g0m2 + g1*ic1eq + g2*ic2eq
t2 = vlow*g3m0 + vband*g3m1 + vhigh*g3m2 + g4*ic1eq + g5*ic2eq
vc1 = t1 + ic1eq (not needed)
```

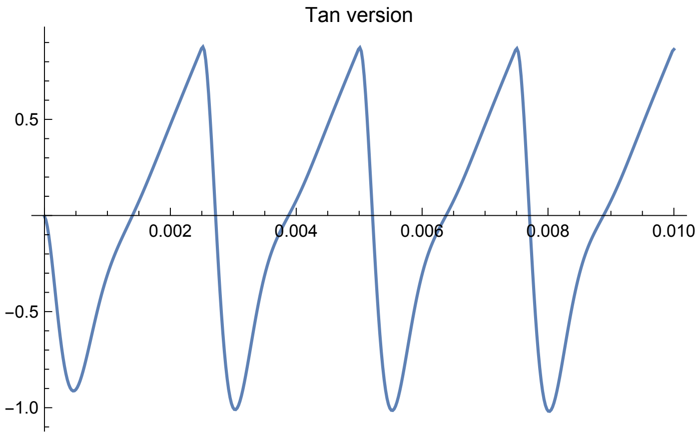
```
vc2 = t2 + ic2eq
ic1eq = ic1eq + 2.0*t1
ic2eq = ic2eq + 2.0*t2
```

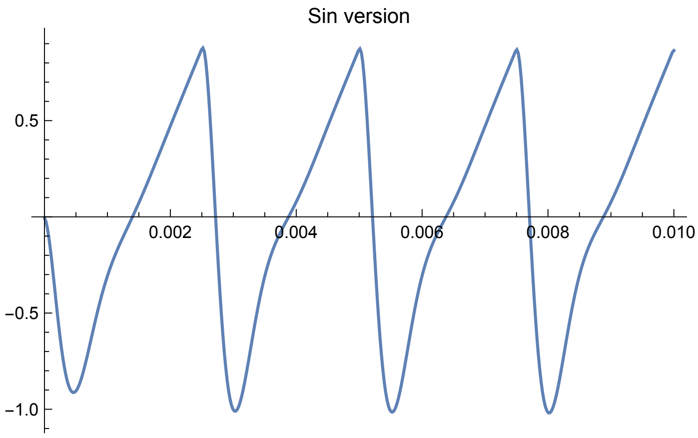
Implementation check against tan version, mixing for low, band, high

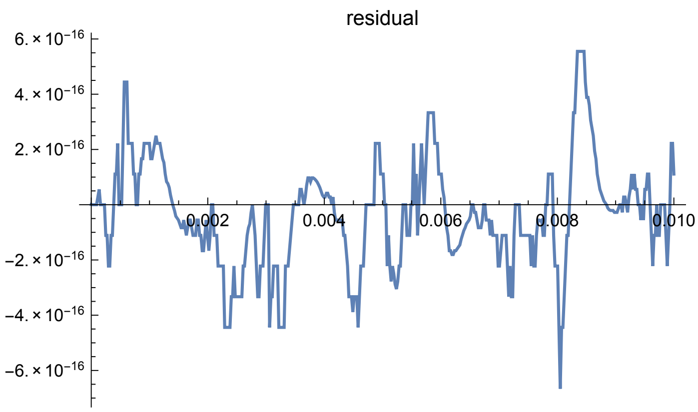
```
CalcCoeff1[cutoff_, res_, mixlow_, mixband_, mixhigh_, sr_] := Block[{w, nrm},
   m0 = mixlow;
   m1 = mixband;
   m2 = mixhigh;
   w = cutoff / sr;
   k = res;
   g = Tan[π w];

  ];
CalcCoeff2[cutoff_, res_, mixlow_, mixband_, mixhigh_, sr_] :=
  Block[{w, nrm, s1, s2, s2n, ssn},
   m0 = mixlow;
   m1 = mixband;
   m2 = mixhigh;
   w = cutoff / sr;
   k = res;
   s1 = Sin[π w];
   s2 = Sin[2 π w];
   w = π * cutoff / sr;
   s1 = Sin[w];
   s2 = Sin[2 * w];
   nrm = 1 / (2 + (2 - k) * s2);
   s1n = 2 * s1 * s1 * nrm;
   s2n = s2 * nrm;
   g0 = m0 * ((1 - k) * s1n + s2n) + m1 * (-s1n - s2n) + m2 * (s1n + (1 - k) * s2n);
   g1 = -s1n - s2n;
   g2 = -k * s2n;
   g3 = m0 * (s1n) + m1 * (s2n) + m2 * (-s1n - (2 - k) * s2n);
   g4 = s2n;
   g5 = -s1n - (1 - k) * s2n;
  ];
Tick1[t_, v0_] := Block[{v1, v2, t1, t2, vout, vc1, vc2},
   vc1 = 1/((1 + g)^2 - g k)
     (-g ic2eq k + ic1eq (1 + g - g k) + g ((1 + g - g k) m0 - (1 + g) m1 + (1 + g - k) m2) v0);
   vc2 = 1/((1 + g)^2 - g k) (ic2eq + g (ic1eq + ic2eq + (g m0 + m1 + (-2 - g + k) m2) v0));
   ic1eq = 2 vc1 - ic1eq;
   ic2eq = 2 vc2 - ic2eq;
   vout = m2 v0 + vc2;
   Return[{t, v0, vout}]
  ];
Tick2[t_, v0_] := Block[{v1, v2, t0, tc1, t1, t2, vout, vc1, vc2},
   t1 = g0 * v0 + g1 * ic1eq + g2 * ic2eq;
   t2 = g3 * v0 + g4 * ic1eq + g5 * ic2eq;
   vc1 = ic1eq + t1;
   vc2 = ic2eq + t2;
   ic1eq = ic1eq + 2.0 * t1;
   ic2eq = ic2eq + 2.0 * t2;
   vout = m2 v0 + vc2;
```

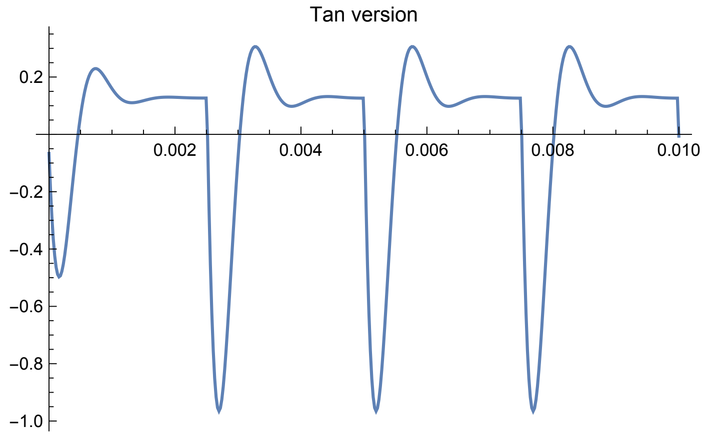
```
    Return[{t, v0, vout}]
   ];

sr = 44100.0;
h = 1.0 / sr;
cutoff = 1000.0;
res = 1.0;

MySaw[x_] := 2 (x - Floor[x] - 0.5);
MyOsc[x_] := 1 MySaw[400 x];

mixlow = 1;
mixband = 0;
mixhigh = 0;
CalcCoeff1[cutoff, res, mixlow, mixband, mixhigh, sr];
CalcCoeff2[cutoff, res, mixlow, mixband, mixhigh, sr];
ic1eq = ic2eq = 0;
tp1 = Table[Tick1[t, MyOsc[t]] , {t, 0, 0.01, h}];
ic1eq = ic2eq = 0;
tp2 = Table[Tick2[t, MyOsc[t]] , {t, 0, 0.01, h}];
ListPlot[Table[tp1[[All, {1, i}]], {i, 3, 3}],
 Joined → True, PlotRange → All, PlotLabel → "Tan version"]
ListPlot[Table[tp2[[All, {1, i}]], {i, 3, 3}],
 Joined → True, PlotRange → All, PlotLabel → "Sin version"]
tp3 = Table[Flatten[{tp1[[j, 1]], tp1[[j, 2]],
      Table[tp1[[j, i]] - tp2[[j, i]], {i, 3, 3}]}], {j, 1, Length[tp1]}];
ListPlot[Table[tp3[[All, {1, i}]], {i, 3, 3}], Joined → True,
 PlotRange → All, PlotLabel → "residual"]

mixlow = 0;
mixband = 1;
mixhigh = 0;
CalcCoeff1[cutoff, res, mixlow, mixband, mixhigh, sr];
CalcCoeff2[cutoff, res, mixlow, mixband, mixhigh, sr];
ic1eq = ic2eq = 0;
tp1 = Table[Tick1[t, MyOsc[t]] , {t, 0, 0.01, h}];
ic1eq = ic2eq = 0;
tp2 = Table[Tick2[t, MyOsc[t]] , {t, 0, 0.01, h}];
ListPlot[Table[tp1[[All, {1, i}]], {i, 3, 3}],
 Joined → True, PlotRange → All, PlotLabel → "Tan version"]
ListPlot[Table[tp2[[All, {1, i}]], {i, 3, 3}],
 Joined → True, PlotRange → All, PlotLabel → "Sin version"]
tp3 = Table[Flatten[{tp1[[j, 1]], tp1[[j, 2]],
      Table[tp1[[j, i]] - tp2[[j, i]], {i, 3, 3}]}], {j, 1, Length[tp1]}];
ListPlot[Table[tp3[[All, {1, i}]], {i, 3, 3}], Joined → True,
 PlotRange → All, PlotLabel → "residual"]

mixlow = 0;
mixband = 0;
mixhigh = 1;
CalcCoeff1[cutoff, res, mixlow, mixband, mixhigh, sr];
CalcCoeff2[cutoff, res, mixlow, mixband, mixhigh, sr];
ic1eq = ic2eq = 0;
tp1 = Table[Tick1[t, MyOsc[t]] , {t, 0, 0.01, h}];
ic1eq = ic2eq = 0;
tp2 = Table[Tick2[t, MyOsc[t]] , {t, 0, 0.01, h}];
ListPlot[Table[tp1[[All, {1, i}]], {i, 3, 3}],
 Joined → True, PlotRange → All, PlotLabel → "Tan version"]
ListPlot[Table[tp2[[All, {1, i}]], {i, 3, 3}],
```

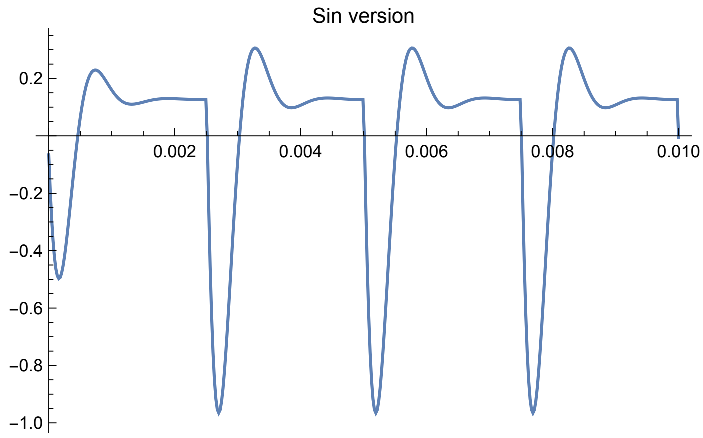
```
 Joined → True, PlotRange → All, PlotLabel → "Sin version"]
tp3 = Table[Flatten[{tp1[[j, 1]], tp1[[j, 2]],
     Table[tp1[[j, i]] - tp2[[j, i]], {i, 3, 3}]}], {j, 1, Length[tp1]}];
ListPlot[Table[tp3[[All, {1, i}]], {i, 3, 3}], Joined → True,
 PlotRange → All, PlotLabel → "residual"]
```

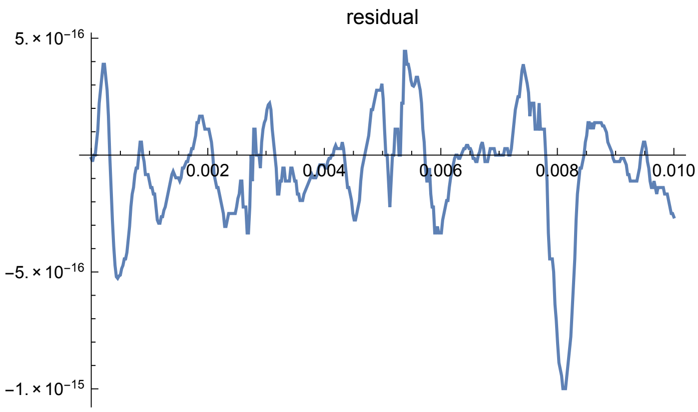


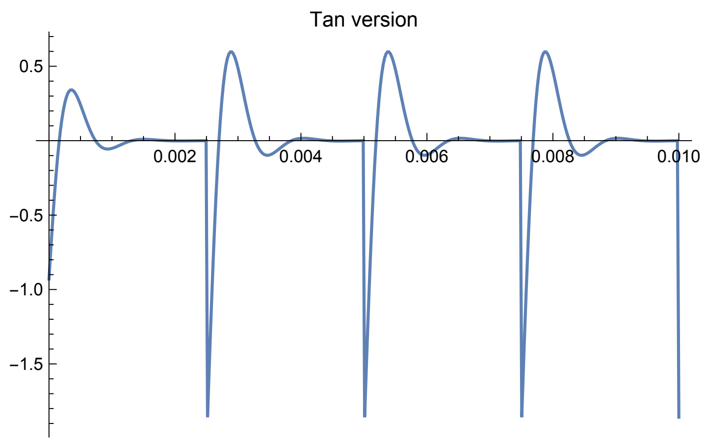


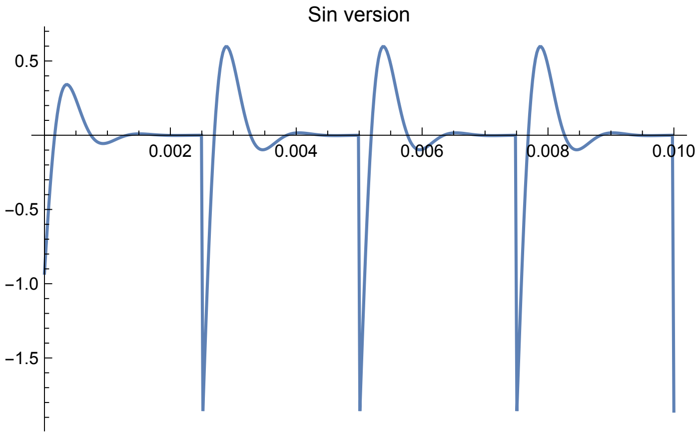


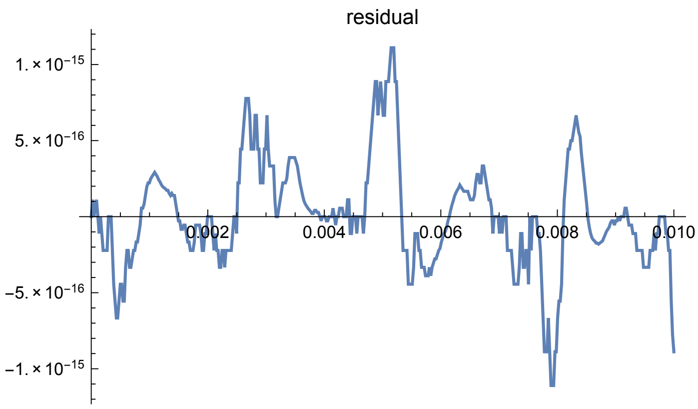

Tan version
0.2
0.002
0.004
0.006
0.008
0.010
−0.2
−0.4
−0.6
−0.8
−1.0


Sin version
0.2
0.002
0.004
0.006
0.008
0.010
−0.2
−0.4
−0.6
−0.8
−1.0


residual
5. × 10−16
0.002
0.004
0.006
0.008
0.010
−5. × 10−16
−1. × 10−15


Tan version
0.5
0.002
0.004
0.006
0.008
0.010
−0.5
−1.0
−1.5


Sin version
0.5
−0.5
−1.0
−1.5
0.002
0.004
0.006
0.008
0.010


residual
1. × 10−15
5. × 10−16
−5. × 10−16
−1. × 10−15
0.002
0.004
0.006
0.008
0.010

Solve for the input mixing SKF to show relation to regular DF1 coefficients

```
Remove["Global`*"]
eqn =
  {va1 == (m0 v0 - m1 v0 - v1), va2 == (m1 v0 + v1 - v2), vc1 == (v1 - (2 - 2 k) v2 + m2 v0),
   vc2 == (v2 - m2 v0), vc1 == ic1eq + g va1, vc2 == ic2eq + g va2,
   ic1eq == (2 vc1 - ic1eq) z^-1, ic2eq == (2 vc2 - ic2eq) z^-1, hz == v2 / v0};
hz = hz /. Solve[eqn, {hz}, {v0, v1, v2, ic1eq, ic2eq, va1, va2, vc1, vc2}][[
    1]] // FullSimplify
skfa = Reverse[CoefficientList[Numerator[hz], z]];
skfb = Reverse[CoefficientList[Denominator[hz], z]];
skfa /= skfb[[1]]
skfb /= skfb[[1]]
```

$$hzskf = \frac{skfa[[1]] + skfa[[2]]\, z^{-1} + skfa[[3]]\, z^{-2}}{1 + skfb[[2]]\, z^{-1} + skfb[[3]]\, z^{-2}}$$

$$hzdf1 = \frac{a0 + a1\, z^{-1} + a2\, z^{-2}}{1 - b1\, z^{-1} - b2\, z^{-2}}$$

```
slndf1 = FullSimplify[Solve[{a0 == skfa[[1]], a1 == skfa[[2]],
    a2 == skfa[[3]], b1 == skfb[[2]], b2 == skfb[[3]]}, {g, k, m0, m1, m2}]]
```

$$\frac{m2\,(-1+z)^2 + g^2\, m0\,(1+z)^2 + g\, m1\,(-1+z^2)}{(-1+z)^2 + g^2\,(1+z)^2 + 2\, g\, k\,(-1+z^2)}$$

$$\left\{\frac{g^2 m0 + g\, m1 + m2}{1+g^2+2 g k}, \frac{2 g^2 m0 - 2 m2}{1+g^2+2 g k}, \frac{g^2 m0 - g\, m1 + m2}{1+g^2+2 g k}\right\}$$

$$\left\{1, \frac{-2+2 g^2}{1+g^2+2 g k}, \frac{1+g^2-2 g k}{1+g^2+2 g k}\right\}$$

$$\frac{\frac{g^2 m0+g\, m1+m2}{1+g^2+2 g k} + \frac{g^2 m0-g\, m1+m2}{(1+g^2+2 g k)\, z^2} + \frac{2 g^2 m0-2 m2}{(1+g^2+2 g k)\, z}}{1 + \frac{1+g^2-2 g k}{(1+g^2+2 g k)\, z^2} + \frac{-2+2 g^2}{(1+g^2+2 g k)\, z}}$$

$$\frac{a0 + \frac{a2}{z^2} + \frac{a1}{z}}{1 - \frac{b2}{z^2} - \frac{b1}{z}}$$

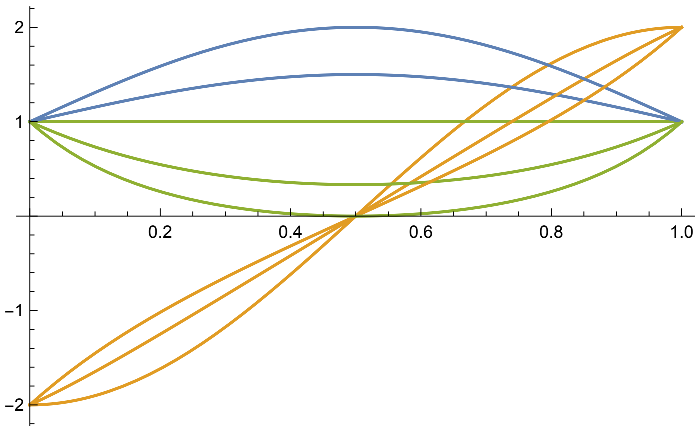
$$\left\{\left\{g \to -\frac{\sqrt{-1-b1-b2}}{\sqrt{-1+b1-b2}}, k \to \frac{1-b2}{\sqrt{-1-b1-b2}\,\sqrt{-1+b1-b2}}, m0 \to \frac{a0+a1+a2}{1+b1+b2}, m1 \to \frac{2\,(a0-a2)}{\sqrt{-1-b1-b2}\,\sqrt{-1+b1-b2}}, m2 \to \frac{a0-a1+a2}{1-b1+b2}\right\},\right.$$
$$\left.\left\{g \to \frac{\sqrt{-1-b1-b2}}{\sqrt{-1+b1-b2}}, k \to \frac{-1+b2}{\sqrt{-1-b1-b2}\,\sqrt{-1+b1-b2}}, m0 \to \frac{a0+a1+a2}{1+b1+b2}, m1 \to -\frac{2\,(a0-a2)}{\sqrt{-1-b1-b2}\,\sqrt{-1+b1-b2}}, m2 \to \frac{a0-a1+a2}{1-b1+b2}\right\}\right\}$$

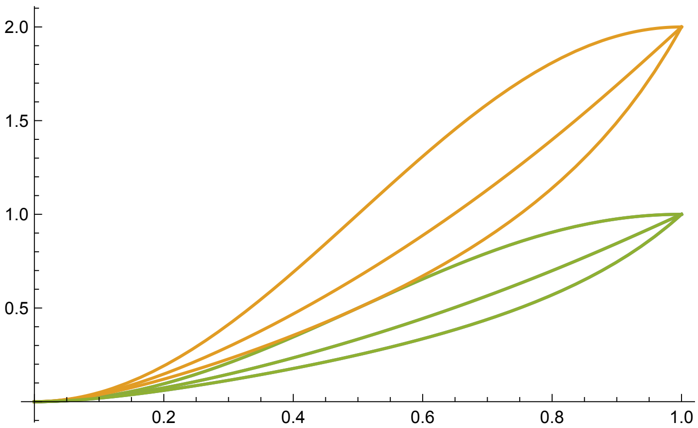
Calculate g, k, m0, m1, m2 for some specific DF1 coefficients.

This is the completely wrong way around to think of this stuff since you should be using the continuous analog filter prototypes since the equations are soooo much easier, as is seen by the results of the coefficients, they are fairly elegant and make sense, where the DF1 coeffs are completely abastract

DF1 low pass

```
cw = Cos[π w];
sw = Sin[π w];
alpha = k * sw / 2;

b0 = 1 + alpha;
b1 = (-2 * cw) / b0;
b2 = (1 - alpha) / b0;
a0 = 1 / 2 (1 - cw) / b0;
a1 = (1 - cw) / b0;
a2 = 1 / 2 (1 - cw) / b0;

Show[Table[Plot[{b0, b1, b2}, {w, 0, 1}, PlotRange → All], {k, 0, 2}]]
Show[Table[Plot[{a0, a1, a2}, {w, 0, 1}, PlotRange → All], {k, 0, 2}]]

sgkm = FullSimplify[slndf1]
SqrSqrtSimplify[eqn_] :=
 FullSimplify[Sqrt[FullSimplify[(eqn)^2]], w > 0 && w < 1 && k > 0]
Table[{g → SqrSqrtSimplify[g /. sgkm[[i]]],
  k → (-1)^(i+1) SqrSqrtSimplify[k /. sgkm[[i]]], m0 → (m0 /. sgkm[[i]]),
  m1 → (-1)^i SqrSqrtSimplify[m1 /. sgkm[[i]]], m2 → (m2 /. sgkm[[i]])}, {i, 1, 2}]
```





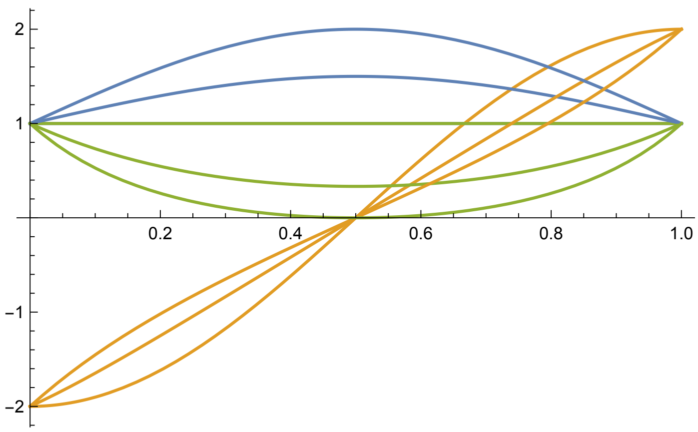
$$\left\{\left\{g\to-\frac{\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ k\to-\frac{k\,\text{Cot}\left[\frac{\pi w}{2}\right]\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{2\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ m0\to1,\ m1\to0,\ m2\to0\right\},\right.$$

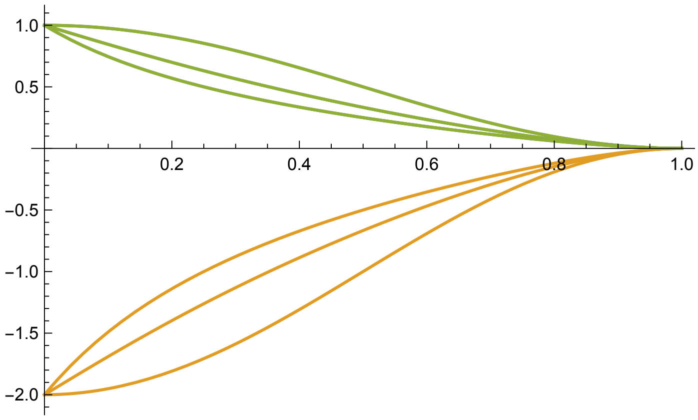
$$\left.\left\{g\to\frac{\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ k\to\frac{k\,\text{Cot}\left[\frac{\pi w}{2}\right]\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{2\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ m0\to1,\ m1\to0,\ m2\to0\right\}\right\}$$

$$\left\{\left\{g\to\text{Tan}\left[\frac{\pi w}{2}\right],\ k\to\frac{k}{2},\ m0\to1,\ m1\to0,\ m2\to0\right\},\ \left\{g\to\text{Tan}\left[\frac{\pi w}{2}\right],\ k\to-\frac{k}{2},\ m0\to1,\ m1\to0,\ m2\to0\right\}\right\}$$

DF1 high pass

```
cw = Cos[π w];
sw = Sin[π w];
alpha = k * sw / 2;

b0 = 1 + alpha;
b1 = (-2 * cw) / b0;
b2 = (1 - alpha) / b0;
a0 = 1 / 2 (1 + cw) / b0;
a1 = - (1 + cw) / b0;
a2 = 1 / 2 (1 + cw) / b0;


Show[Table[Plot[{b0, b1, b2}, {w, 0, 1}, PlotRange → All], {k, 0, 2}]]
Show[Table[Plot[{a0, a1, a2}, {w, 0, 1}, PlotRange → All], {k, 0, 2}]]

sgkm = FullSimplify[slndf1]
SqrSqrtSimplify[eqn_] :=
 FullSimplify[Sqrt[FullSimplify[(eqn)^2]], w > 0 && w < 1 && k > 0]
Table[{g → SqrSqrtSimplify[g /. sgkm[[i]]],
   k → (-1)^(i+1) SqrSqrtSimplify[k /. sgkm[[i]]], m0 → (m0 /. sgkm[[i]]),
   m1 → (-1)^i SqrSqrtSimplify[m1 /. sgkm[[i]]], m2 → (m2 /. sgkm[[i]])}, {i, 1, 2}]
```





$$\left\{\left\{g\to -\frac{\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ k\to -\frac{k\,\text{Cot}\left[\frac{\pi w}{2}\right]\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{2\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ m0\to 0,\ m1\to 0,\ m2\to 1\right\},\right.$$

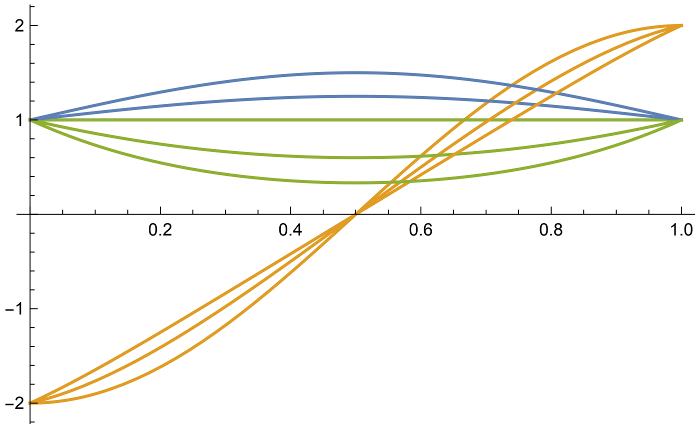
$$\left.\left\{g\to \frac{\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ k\to \frac{k\,\text{Cot}\left[\frac{\pi w}{2}\right]\sqrt{\frac{-1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}}{2\sqrt{-\frac{1+\text{Cos}[\pi w]}{2+k\,\text{Sin}[\pi w]}}},\ m0\to 0,\ m1\to 0,\ m2\to 1\right\}\right\}$$

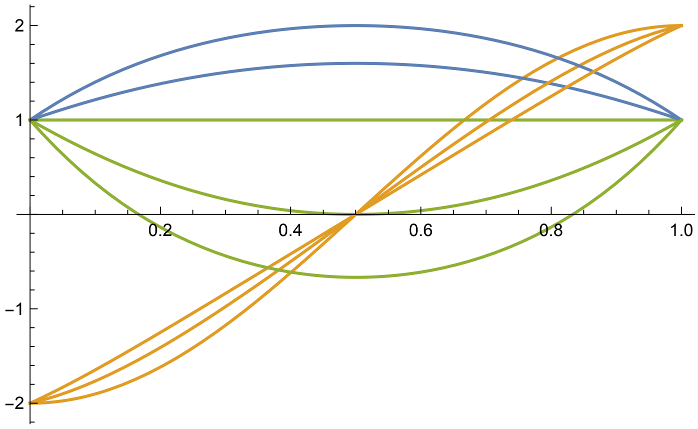
$$\left\{\left\{g\to \text{Tan}\left[\frac{\pi w}{2}\right],\ k\to \frac{k}{2},\ m0\to 0,\ m1\to 0,\ m2\to 1\right\},\right.$$

$$\left.\left\{g\to \text{Tan}\left[\frac{\pi w}{2}\right],\ k\to -\frac{k}{2},\ m0\to 0,\ m1\to 0,\ m2\to 1\right\}\right\}$$

DF1 bell

```
cw = Cos[π w];
sw = Sin[π w];
alpha = k * sw / 2;

b0 = 1 + alpha / A;
b1 = (-2 * cw) / b0;
b2 = (1 - alpha / A) / b0;
a0 = (1 + alpha * A) / b0;
a1 = (-2 * cw) / b0;
a2 = (1 - alpha * A) / b0;


Show[Table[Plot[{b0 /. {A → 2}, b1 /. {A → 2}, b2 /. {A → 2}},
    {w, 0, 1}, PlotRange → All], {k, 0, 2}]]
Show[Table[Plot[{a0 /. {A → 2}, a1 /. {A → 2}, a2 /. {A → 2}},
    {w, 0, 1}, PlotRange → All], {k, 0, 2}]]

sgkm = FullSimplify[slndf1]
SqrSqrtSimplify[eqn_] :=
 FullSimplify[Sqrt[FullSimplify[(eqn)^2]], w > 0 && w < 1 && k > 0 && A > 0]
Table[{g → SqrSqrtSimplify[g /. sgkm[[i]]],
  k → (-1)^i SqrSqrtSimplify[k /. sgkm[[i]]], m0 → (m0 /. sgkm[[i]]),
  m1 → (-1)^i SqrSqrtSimplify[m1 /. sgkm[[i]]], m2 → (m2 /. sgkm[[i]])}, {i, 1, 2}]
```





$$\left\{\left\{g\to-\frac{\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}}{\sqrt{-\frac{A\,(1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}},\ k\to\frac{k\,\text{Sin}[\pi\,w]}{2\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}\,\sqrt{-\frac{A\,(1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}\,\left(2\,A+k\,\text{Sin}[\pi\,w]\right)},\right.\right.$$

$$\left.m0\to1,\ m1\to\frac{A\,k\,\text{Sin}[\pi\,w]\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}}{(-1+\text{Cos}[\pi\,w])\,\sqrt{-\frac{A\,(1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}},\ m2\to1\right\},$$

$$\left\{g\to\frac{\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}}{\sqrt{-\frac{A\,(1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}},\ k\to\frac{k\,\text{Sin}[\pi\,w]\,\sqrt{-\frac{A\,(1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}}{2\,A\,(1+\text{Cos}[\pi\,w])\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}},\right.$$

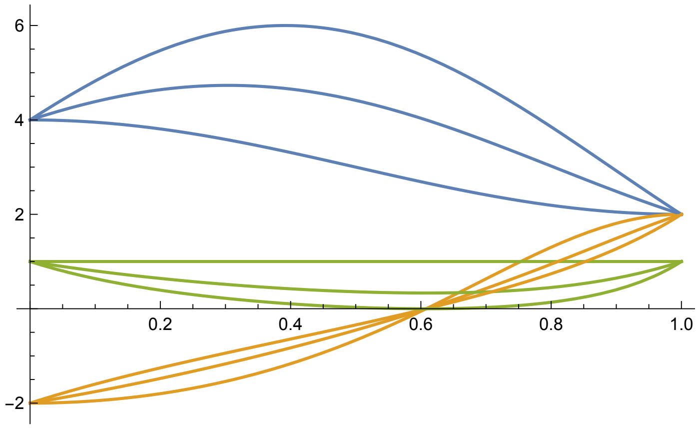
$$\left.\left.m0\to1,\ m1\to\frac{A\,k\,\text{Cot}\left[\frac{\pi\,w}{2}\right]\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}}{\sqrt{-\frac{A\,(1+\text{Cos}[\pi\,w])}{2\,A+k\,\text{Sin}[\pi\,w]}}},\ m2\to1\right\}\right\}$$

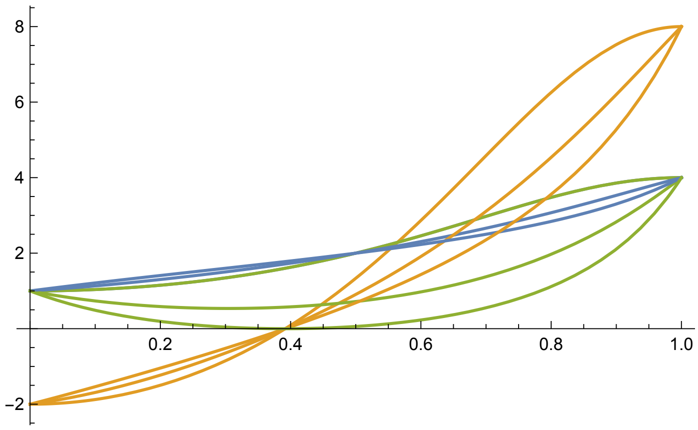
$$\left\{\left\{g\to\text{Tan}\left[\frac{\pi\,w}{2}\right],\ k\to-\frac{k}{2\,A},\ m0\to1,\ m1\to-A\,k,\ m2\to1\right\},\right.$$

$$\left.\left\{g\to\text{Tan}\left[\frac{\pi\,w}{2}\right],\ k\to\frac{k}{2\,A},\ m0\to1,\ m1\to A\,k,\ m2\to1\right\}\right\}$$

DF1 low shelf

```
cw = Cos[π w];
sw = Sin[π w];
alpha = k * sw / 2;

sqrtA = Sqrt[A];
b0 = ((A + 1) + (A - 1) * cw + 2 * sqrtA * alpha);
b1 = (-2 * ((A - 1) + (A + 1) * cw)) / b0;
b2 = ((A + 1) + (A - 1) * cw - 2 * sqrtA * alpha) / b0;
a0 = (A * ((A + 1) - (A - 1) * cw + 2 * sqrtA * alpha)) / b0;
a1 = (2 * A * ((A - 1) - (A + 1) * cw)) / b0;
a2 = (A * ((A + 1) - (A - 1) * cw - 2 * sqrtA * alpha)) / b0;


Show[Table[Plot[{b0 /. {A → 2}, b1 /. {A → 2}, b2 /. {A → 2}},
    {w, 0, 1}, PlotRange → All], {k, 0, 2}]]
Show[Table[Plot[{a0 /. {A → 2}, a1 /. {A → 2}, a2 /. {A → 2}},
    {w, 0, 1}, PlotRange → All], {k, 0, 2}]]

sgkm = FullSimplify[slndf1]
SqrSqrtSimplify[eqn_] :=
 FullSimplify[Sqrt[FullSimplify[(eqn)^2]], w > 0 && w < 1 && k > 0 && A > 0]
Table[{g → SqrSqrtSimplify[g /. sgkm[[i]]],
   k → (-1)^i SqrSqrtSimplify[k /. sgkm[[i]]], m0 → (m0 /. sgkm[[i]]),
   m1 → (-1)^i SqrSqrtSimplify[m1 /. sgkm[[i]]], m2 → (m2 /. sgkm[[i]])}, {i, 1, 2}]
```





$$\left\{\left\{g \to -\frac{\sqrt{\frac{-1+\text{Cos}[\pi w]}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}}{\sqrt{-\frac{A\,(1+\text{Cos}[\pi w])}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}},\right.\right.$$

$$k \to \frac{\sqrt{A}\,k\,\text{Sin}[\pi w]\sqrt{\frac{-1+\text{Cos}[\pi w]}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}}{2\,(-1+\text{Cos}[\pi w])\sqrt{-\frac{A\,(1+\text{Cos}[\pi w])}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}},\ m0 \to A^2,$$

$$\left. m1 \to \frac{A^{3/2}\,k\,\text{Sin}[\pi w]\sqrt{\frac{-1+\text{Cos}[\pi w]}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}}{(-1+\text{Cos}[\pi w])\sqrt{-\frac{A\,(1+\text{Cos}[\pi w])}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}},\ m2 \to 1\right\},$$

$$\left\{g \to \frac{\sqrt{\frac{-1+\text{Cos}[\pi w]}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}}{\sqrt{-\frac{A\,(1+\text{Cos}[\pi w])}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}},\right.$$

$$k \to -\frac{\sqrt{A}\,k\,\text{Sin}[\pi w]\sqrt{\frac{-1+\text{Cos}[\pi w]}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}}{2\,(-1+\text{Cos}[\pi w])\sqrt{-\frac{A\,(1+\text{Cos}[\pi w])}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}},\ m0 \to A^2,$$

$$\left.\left. m1 \to -\frac{A^{3/2}\,k\,\text{Sin}[\pi w]\sqrt{\frac{-1+\text{Cos}[\pi w]}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}}{(-1+\text{Cos}[\pi w])\sqrt{-\frac{A\,(1+\text{Cos}[\pi w])}{1+A+(-1+A)\,\text{Cos}[\pi w]+\sqrt{A}\,k\,\text{Sin}[\pi w]}}},\ m2 \to 1\right\}\right\}$$

$$\left\{\left\{g \to \frac{\text{Tan}\left[\frac{\pi w}{2}\right]}{\sqrt{A}},\ k \to -\frac{k}{2},\ m0 \to A^2,\ m1 \to -A\,k,\ m2 \to 1\right\},\right.$$

$$\left.\left\{g \to \frac{\text{Tan}\left[\frac{\pi w}{2}\right]}{\sqrt{A}},\ k \to \frac{k}{2},\ m0 \to A^2,\ m1 \to A\,k,\ m2 \to 1\right\}\right\}$$

DF1 high shelf

```
cw = Cos[π w];
sw = Sin[π w];
alpha = k * sw / 2;

sqrtA = Sqrt[A];
b0 = (A + 1) - (A - 1) * cw + 2 * sqrtA * alpha
b1 = 2 * ((A - 1) - (A + 1) * cw) / b0
b2 = ((A + 1) - (A - 1) * cw - 2 * sqrtA * alpha) / b0
a0 = A * ((A + 1) + (A - 1) * cw + 2 * sqrtA * alpha) / b0
a1 = -2 * A * ((A - 1) + (A + 1) * cw) / b0
a2 = A * ((A + 1) + (A - 1) * cw - 2 * sqrtA * alpha) / b0


Show[Table[Plot[{b0 /. {A → 2}, b1 /. {A → 2}, b2 /. {A → 2}},
    {w, 0, 1}, PlotRange → All], {k, 0, 2}]]
Show[Table[Plot[{a0 /. {A → 2}, a1 /. {A → 2}, a2 /. {A → 2}},
    {w, 0, 1}, PlotRange → All], {k, 0, 2}]]

sgkm = FullSimplify[slndf1]
SqrSqrtSimplify[eqn_] :=
 FullSimplify[Sqrt[FullSimplify[(eqn)^2]], w > 0 && w < 1 && k > 0 && A > 0]
Table[{g → SqrSqrtSimplify[g /. sgkm[[i]]],
   k → (-1)^i SqrSqrtSimplify[k /. sgkm[[i]]], m0 → (m0 /. sgkm[[i]]),
   m1 → (-1)^i SqrSqrtSimplify[m1 /. sgkm[[i]]], m2 → (m2 /. sgkm[[i]])}, {i, 1, 2}]
```

$$1 + A - (-1 + A)\, \mathrm{Cos}[\pi w] + \sqrt{A}\, k\, \mathrm{Sin}[\pi w]$$

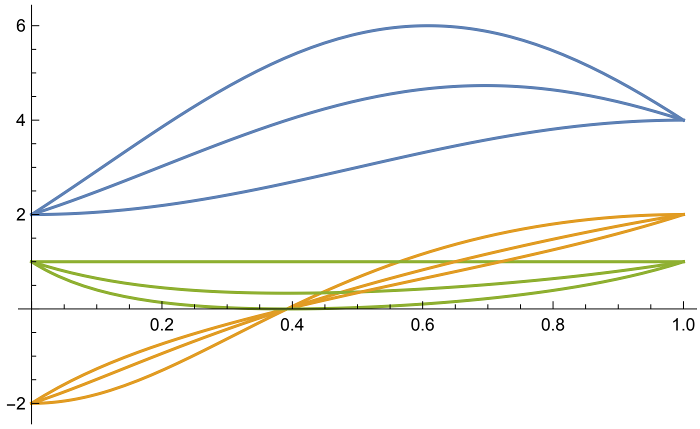
$$\frac{2\,(-1 + A - (1 + A)\, \mathrm{Cos}[\pi w])}{1 + A - (-1 + A)\, \mathrm{Cos}[\pi w] + \sqrt{A}\, k\, \mathrm{Sin}[\pi w]}$$

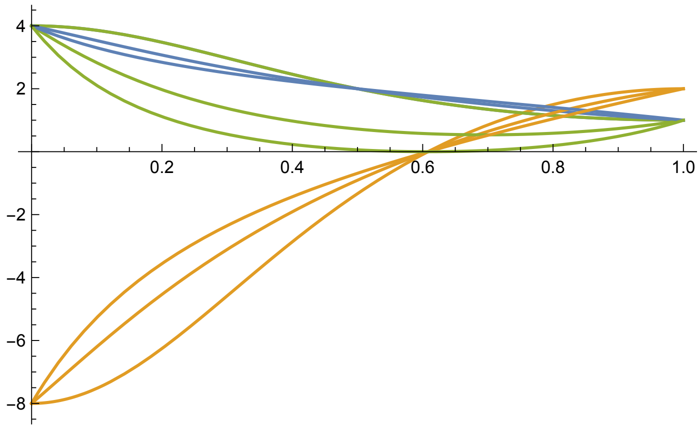
$$\frac{1 + A - (-1 + A)\, \mathrm{Cos}[\pi w] - \sqrt{A}\, k\, \mathrm{Sin}[\pi w]}{1 + A - (-1 + A)\, \mathrm{Cos}[\pi w] + \sqrt{A}\, k\, \mathrm{Sin}[\pi w]}$$

$$\frac{A\left(1 + A + (-1 + A)\, \mathrm{Cos}[\pi w] + \sqrt{A}\, k\, \mathrm{Sin}[\pi w]\right)}{1 + A - (-1 + A)\, \mathrm{Cos}[\pi w] + \sqrt{A}\, k\, \mathrm{Sin}[\pi w]}$$

$$-\frac{2\,A\,(-1 + A + (1 + A)\, \mathrm{Cos}[\pi w])}{1 + A - (-1 + A)\, \mathrm{Cos}[\pi w] + \sqrt{A}\, k\, \mathrm{Sin}[\pi w]}$$

$$\frac{A\left(1 + A + (-1 + A)\, \mathrm{Cos}[\pi w] - \sqrt{A}\, k\, \mathrm{Sin}[\pi w]\right)}{1 + A - (-1 + A)\, \mathrm{Cos}[\pi w] + \sqrt{A}\, k\, \mathrm{Sin}[\pi w]}$$


6
4
2
-2
0.2
0.4
0.6
0.8
1.0


4
2
-2
-4
-6
-8
0.2
0.4
0.6
0.8
1.0

$$\left\{\left\{g \to -\frac{\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}}{\sqrt{-\frac{1+\text{Cos}[\pi\,w]}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}},\right.\right.$$

$$k \to -\frac{\sqrt{A}\,k\,\text{Sin}[\pi\,w]\,\sqrt{-\frac{1+\text{Cos}[\pi\,w]}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}}{2\,(1+\text{Cos}[\pi\,w])\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}},\ m0 \to 1,$$

$$\left.m1 \to -\frac{A^{3/2}\,k\,\text{Sin}[\pi\,w]\,\sqrt{-\frac{1+\text{Cos}[\pi\,w]}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}}{(1+\text{Cos}[\pi\,w])\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}},\ m2 \to A^2\right\},$$

$$\left\{g \to \frac{\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}}{\sqrt{-\frac{1+\text{Cos}[\pi\,w]}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}},\ k \to \frac{\sqrt{A}\,k\,\text{Sin}[\pi\,w]\,\sqrt{-\frac{1+\text{Cos}[\pi\,w]}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}}{2\,(1+\text{Cos}[\pi\,w])\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}},\right.$$

$$\left.\left.m0 \to 1,\ m1 \to \frac{A^{3/2}\,k\,\text{Sin}[\pi\,w]\,\sqrt{-\frac{1+\text{Cos}[\pi\,w]}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}}{(1+\text{Cos}[\pi\,w])\,\sqrt{\frac{A\,(-1+\text{Cos}[\pi\,w])}{1+A-(-1+A)\,\text{Cos}[\pi\,w]+\sqrt{A}\,k\,\text{Sin}[\pi\,w]}}},\ m2 \to A^2\right\}\right\}$$

$$\left\{\left\{g \to \sqrt{A}\,\text{Tan}\left[\frac{\pi\,w}{2}\right],\ k \to -\frac{k}{2},\ m0 \to 1,\ m1 \to -A\,k,\ m2 \to A^2\right\},\right.$$
$$\left.\left\{g \to \sqrt{A}\,\text{Tan}\left[\frac{\pi\,w}{2}\right],\ k \to \frac{k}{2},\ m0 \to 1,\ m1 \to A\,k,\ m2 \to A^2\right\}\right\}$$